

Logic

Logic is, basically, the study of valid reasoning. When searching the internet, we use Boolean logic – terms like “and” and “or” – to help us find specific web pages that fit in the sets we are interested in. After exploring this form of logic, we will look at logical arguments and how we can determine the validity of a claim.

Boolean Logic

We can often classify items as belonging to sets. If you went the library to search for a book and they asked you to express your search using unions, intersections, and complements of sets, that would feel a little strange. Instead, we typically using words like “and”, “or”, and “not” to connect our keywords together to form a search. These words, which form the basis of **Boolean logic**, are directly related to our set operations.

Boolean Logic

Boolean logic combines multiple statements that are either true or false into an expression that is either true or false.

In connection to sets, a search is true if the element is part of the set.

Suppose M is the set of all mystery books, and C is the set of all comedy books. If we search for “mystery”, we are looking for all the books that are an element of the set M ; the search is true for books that are in the set.

When we search for “mystery *and* comedy”, we are looking for a book that is an element of both sets, in the intersection. If we were to search for “mystery *or* comedy”, we are looking for a book that is a mystery, a comedy, or both, which is the union of the sets. If we searched for “*not* comedy”, we are looking for any book in the library that is not a comedy, the complement of the set C .

Connection to Set Operations

A and B	elements in the intersection $A \cap B$
A or B	elements in the union $A \cup B$
Not A	elements in the complement A^c

Notice here that *or* is not exclusive. This is a difference between the Boolean logic use of the word and common everyday use. When your significant other asks “do you want to go to the park *or* the movies?” they usually are proposing an exclusive choice – one option or the other, but not both. In Boolean logic, the *or* is not exclusive – more like being asked at a restaurant “would you like fries *or* a drink with that?” Answering “both, please” is an acceptable answer.

Example 1

Suppose we are searching a library database for Mexican universities. Express a reasonable search using Boolean logic.

We could start with the search “Mexico *and* university”, but would be likely to find results for the U.S. state New Mexico. To account for this, we could revise our search to read: Mexico *and* university *not* “New Mexico”

In most internet search engines, it is not necessary to include the word *and*; the search engine assumes that if you provide two keywords you are looking for both. In Google’s search, the keyword *or* has to be capitalized as OR, and a negative sign in front of a word is used to indicate *not*. Quotes around a phrase indicate that the entire phrase should be looked for. The search from the previous example on Google could be written:
Mexico university -“New Mexico”

Example 2

Describe the numbers that meet the condition:
even and less than 10 and greater than 0

The numbers that satisfy all three requirements are {2, 4, 6, 8}

Sometimes statements made in English can be ambiguous. For this reason, Boolean logic uses parentheses to show precedent, just like in algebraic order of operations.

Example 3

The English phrase “Go to the store and buy me eggs and bagels or cereal” is ambiguous; it is not clear whether the requestor is asking for eggs always along with either bagels or cereal, or whether they’re asking for either the combination of eggs and bagels, or just cereal.

For this reason, using parentheses clarifies the intent:

Eggs and (bagels or cereal) means Option 1: Eggs and bagels, Option 2: Eggs and cereal
(Eggs and bagels) or cereal means Option 1: Eggs and bagels, Option 2: Cereal

Example 4

Describe the numbers that meet the condition:
odd number and less than 20 and greater than 0 and (multiple of 3 or multiple of 5)

The first three conditions limit us to the set {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

The last grouped conditions tell us to find elements of this set that are also either a multiple of 3 or a multiple of 5. This leaves us with the set {3, 5, 9, 15}

Notice that we would have gotten a very different result if we had written
(odd number and less than 20 and greater than 0 and multiple of 3) or multiple of 5

The first grouped set of conditions would give {3, 9, 15}. When combined with the last condition, though, this set expands without limits:
 {3, 5, 9, 15, 20, 25, 30, 35, 40, 45, ...}

Be aware that when a string of conditions is written without grouping symbols, it is often interpreted from the left to right, resulting in the latter interpretation.

Conditionals

Beyond searching, Boolean logic is commonly used in spreadsheet applications like Excel to do conditional calculations. A **statement** is something that is either true or false. A statement like $3 < 5$ is true; a statement like “a rat is a fish” is false. A statement like “ $x < 5$ ” is true for some values of x and false for others. When an action is taken or not depending on the value of a statement, it forms a **conditional**.

Statements and Conditionals

A **statement** is either true or false.

A **conditional** is a compound statement of the form
 “if p then q ” or “if p then q , else s ”.

Example 5

In common language, an example of a conditional statement would be “If it is raining, then we’ll go to the mall. Otherwise we’ll go for a hike.”

The statement “If it is raining” is the condition – this may be true or false for any given day. If the condition is true, then we will follow the first course of action, and go to the mall. If the condition is false, then we will use the alternative, and go for a hike.

Example 6

As mentioned earlier, conditional statements are commonly used in spreadsheet applications like Excel. If Excel, you can enter an expression like
`=IF(A1<2000, A1+1, A1*2)`

Notice that after the IF, there are three parts. The first part is the condition, and the second two are calculations. Excel will look at the value in cell A1 and compare it to 2000. If that condition is true, then the first calculation is used, and 1 is added to the value of A1 and the result is stored. If the condition is false, then the second calculation is used, and A1 is multiplied by 2 and the result is stored.

In other words, this statement is equivalent to saying “If the value of A1 is less than 2000, then add 1 to the value in A1. Otherwise, multiple A1 by 2”

Example 7

The expression `=IF(A1 > 5, 2*A1, 3*A1)` is used. Find the result if A1 is 3, and the result if A1 is 8

This is equivalent to saying

If $A1 > 5$, then calculate $2*A1$. Otherwise, calculate $3*A1$

If A1 is 3, then the condition is false, since $3 > 5$ is not true, so we do the alternate action, and multiple by 3, giving $3*3 = 9$

If A1 is 8, then the condition is true, since $8 > 5$, so we multiply the value by 2, giving $2*8=16$

Example 8

An accountant needs to withhold 15% of income for taxes if the income is below \$30,000, and 20% of income if the income is \$30,000 or more. Write an expression that would calculate the amount to withhold.

Our conditional needs to compare the value to 30,000. If the income is less than 30,000, we need to calculate 15% of the income: $0.15*income$. If the income is more than 30,000, we need to calculate 20% of the income: $0.20*income$.

In words we could write “If $income < 30,000$, then multiply by 0.15, otherwise multiply by 0.20”. In Excel, we would write:

`=IF(A1<30000, 0.15*A1, 0.20*A1)`

As we did earlier, we can create more complex conditions by using the operators *and*, *or*, and *not* to join simpler conditions together.

Example 9

A parent might say to their child “if you clean your room and take out the garbage, then you can have ice cream.”

Here, there are two simpler conditions:

- 1) The child cleaning her room
- 2) The child taking out the garbage

Since these conditions were joined with *and*, then the combined conditional will only be true if both simpler conditions are true; if either chore is not completed then the parent’s condition is not met.

Notice that if the parent had said “if you clean your room *or* take out the garbage, then you can have ice cream”, then the child would only need to complete one chore to meet the condition.

To create the condition “ $A1 < 3000$ and $A1 > 100$ ” in Excel, you would need to enter “AND($A1 < 3000$, $A1 > 100$)”. Likewise, for the condition “ $A1 = 4$ or $A1 = 6$ ” you would enter “OR($A1 = 4$, $A1 = 6$)”

Example 10

In a spreadsheet, cell A1 contains annual income, and A2 contains number of dependents. A certain tax credit applies if someone with no dependents earns less than \$10,000 and has no dependents, or if someone with dependents earns less than \$20,000. Write a rule that describes this.

There are two ways the rule is met:
income is less than 10,000 *and* dependents is 0, *or*
income is less than 20,000 *and* dependents is not 0.

Informally, we could write these as
($A1 < 10000$ *and* $A2 = 0$) *or* ($A1 < 20000$ *and* $A2 > 0$)

Notice that the $A2 > 0$ condition is actually redundant and not necessary, since we’d only be considering that *or* case if the first pair of conditions were not met. So this could be simplified to
($A1 < 10000$ *and* $A2 = 0$) *or* ($A1 < 20000$)

In Excel’s format, we’d write
= IF (OR(AND($A1 < 10000$, $A2 = 0$), $A1 < 20000$), “you qualify”, “you don’t qualify”)

Truth Tables

Because complex Boolean statements can get tricky to think about, we can create a **truth table** to keep track of what truth values for the simple statements make the complex statement true and false

Truth table

A table showing what the resulting truth value of a complex statement is for all the possible truth values for the simple statements.

Example 11

Suppose you’re picking out a new couch, and your significant other says “get a sectional *or* something with a chaise”.

This is a complex statement made of two simpler conditions: “is a sectional”, and “has a chaise”. For simplicity, let’s use S to designate “is a sectional”, and C to designate “has a chaise”. The condition S is true if the couch is a sectional.

A truth table for this would look like this:

S	C	S or C
T	T	T
T	F	T
F	T	T
F	F	F

In the table, T is used for true, and F for false. In the first row, if S is true and C is also true, then the complex statement “ S or C ” is true. This would be a sectional that also has a chaise, which meets our desire.

Remember also that *or* in logic is not exclusive; if the couch has both features, it does meet the condition.

To shorthand our notation further, we’re going to introduce some symbols that are commonly used for *and*, *or*, and *not*.

Symbols

The symbol $\wedge$ is used for *and*: A and B is notated $A \wedge B$.

The symbol $\vee$ is used for *or*: A or B is notated $A \vee B$

The symbol $\sim$ is used for *not*: not A is notated $\sim A$

You can remember the first two symbols by relating them to the shapes for the union and intersection. $A \wedge B$ would be the elements that exist in both sets, in $A \cap B$. Likewise, $A \vee B$ would be the elements that exist in either set, in $A \cup B$.

In the previous example, the truth table was really just summarizing what we already know about how the *or* statement work. The truth tables for the basic *and*, *or*, and *not* statements are shown below.

Basic truth tables

A	B	$A \wedge B$
T	T	T
T	F	F
F	T	F
F	F	F

A	B	$A \vee B$
T	T	T
T	F	T
F	T	T
F	F	F

A	$\sim A$
T	F
F	T

Truth tables really become useful when analyzing more complex Boolean statements.

Example 12

Create a truth table for the statement $A \wedge \sim(B \vee C)$

It helps to work from the inside out when creating truth tables, and create tables for intermediate operations. We start by listing all the possible truth value combinations for A , B , and C . Notice how the first column contains 4 Ts followed by 4 Fs, the second column contains 2 Ts, 2 Fs, then repeats, and the last column alternates. This pattern ensures that all combinations are considered. Along with those initial values, we'll list the truth values for the innermost expression, $B \vee C$.

A	B	C	$B \vee C$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	F

Next we can find the negation of $B \vee C$, working off the $B \vee C$ column we just created.

A	B	C	$B \vee C$	$\sim(B \vee C)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	F
T	F	F	F	T
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	T

Finally, we find the values of A and $\sim(B \vee C)$

A	B	C	$B \vee C$	$\sim(B \vee C)$	$A \wedge \sim(B \vee C)$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	F

It turns out that this complex expression is only true in one case: if A is true, B is false, and C is false.

Try it Now 1

Create a truth table for this statement: $(\sim A \wedge B) \vee \sim B$

When we discussed conditions earlier, we discussed the type where we take an action based on the value of the condition. We are now going to talk about a more general version of a conditional, sometimes called an **implication**.

Implications

Implications are logical conditional sentences stating that a statement p , called the antecedent, implies a consequence q .

Implications are commonly written as $p \rightarrow q$

Implications are similar to the conditional statements we looked at earlier; $p \rightarrow q$ is typically written as “if p then q ”, or “ p therefore q ”. The difference between implications and conditionals is that conditionals we discussed earlier suggest an action – if the condition is true, then we take some action as a result. Implications are a logical statement that suggest that the consequence must logically follow if the antecedent is true.

Example 13

The English statement “If it is raining, then there are clouds in the sky” is a logical implication. It is a valid argument because if the antecedent “it is raining” is true, then the consequence “there are clouds in the sky” must also be true.

Notice that the statement tells us nothing of what to expect if it is not raining. If the antecedent is false, then the implication becomes irrelevant.

Example 14

A friend tells you that “if you upload that picture to Facebook, you’ll lose your job”. There are four possible outcomes:

- 1) You upload the picture and keep your job
- 2) You upload the picture and lose your job
- 3) You don’t upload the picture and keep your job
- 4) You don’t upload the picture and lose your job

There is only one possible case where your friend was lying – the first option where you upload the picture and keep your job. In the last two cases, your friend didn’t say anything about what would happen if you didn’t upload the picture, so you can’t conclude their statement is invalid, even if you didn’t upload the picture and still lost your job.

In traditional logic, an implication is considered valid (true) as long as there are no cases in which the antecedent is true and the consequence is false. It is important to keep in mind that symbolic logic cannot capture all the intricacies of the English language.

Truth values for implications

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example 15

Construct a truth table for the statement $(m \wedge \sim p) \rightarrow r$

We start by constructing a truth table for the antecedent.

m	p	$\sim p$	$m \wedge \sim p$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Now we can build the truth table for the implication

m	p	$\sim p$	$m \wedge \sim p$	r	$(m \wedge \sim p) \rightarrow r$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	T	T
T	T	F	F	F	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	F	F	T

In this case, when m is true, p is false, and r is false, then the antecedent $m \wedge \sim p$ will be true but the consequence false, resulting in a invalid implication; every other case gives a valid implication.

For any implication, there are three related statements, the converse, the inverse, and the contrapositive.

Related Statements

The original implication is	“if p then q ”	$p \rightarrow q$
The converse is:	“if q then p ”	$q \rightarrow p$
The inverse is	“if not p then not q ”	$\sim p \rightarrow \sim q$
The contrapositive is	“if not q then not p ”	$\sim q \rightarrow \sim p$

Example 16

Consider again the valid implication “If it is raining, then there are clouds in the sky”.

The converse would be “If there are clouds in the sky, it is raining.” This is certainly not always true.

The inverse would be “If it is not raining, then there are not clouds in the sky.” Likewise, this is not always true.

The contrapositive would be “If there are not clouds in the sky, then it is not raining.” This statement is valid, and is equivalent to the original implication.

Looking at truth tables, we can see that the original conditional and the contrapositive are logically equivalent, and that the converse and inverse are logically equivalent.

		Implication	Converse	Inverse	Contrapositive
p	q	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

Equivalence

A conditional statement and its contrapositive are logically equivalent.
The converse and inverse of a statement are logically equivalent.

We have now examined several truth-functional connectives, three of which are two-place connectives – conjunction, disjunction, and implication (conditional) – and one of which is a one-place connective (negation). There is one remaining connective that is generally studied in sentential logic, the **biconditional**, which corresponds to the English “if and only if”.

Like the implication, the biconditional is a two-place connective; if we fill the two blanks with statements, the resulting expression is also a statement. For example, we can begin with

the statements “I am happy” and “I am relaxed” and form the compound statement “I am happy if and only if I am relaxed”.

Truth values for the biconditional

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Arguments

A logical argument is a claim that a set of premises support a conclusion. There are two general types of arguments: inductive and deductive arguments.

Argument types

An **inductive** argument uses a collection of specific examples as its premises and uses them to propose a general conclusion.

A **deductive** argument uses a collection of general statements as its premises and uses them to propose a specific situation as the conclusion.

Example 17

The argument “when I went to the store last week I forgot my purse, and when I went today I forgot my purse. I always forget my purse when I go the store” is an inductive argument.

The premises are:

I forgot my purse last week

I forgot my purse today

The conclusion is:

I always forget my purse

Notice that the premises are specific situations, while the conclusion is a general statement. In this case, this is a fairly weak argument, since it is based on only two instances.

Example 18

The argument “every day for the past year, a plane flies over my house at 2pm. A plane will fly over my house every day at 2pm” is a stronger inductive argument, since it is based on a larger set of evidence.

Evaluating inductive arguments

An inductive argument is never able to prove the conclusion true, but it can provide either weak or strong evidence to suggest it may be true.

Many scientific theories, such as the big bang theory, can never be proven. Instead, they are inductive arguments supported by a wide variety of evidence. Usually in science, an idea is considered a hypothesis until it has been well tested, at which point it graduates to being considered a theory. The commonly known scientific theories, like Newton's theory of gravity, have all stood up to years of testing and evidence, though sometimes they need to be adjusted based on new evidence. For gravity, this happened when Einstein proposed the theory of general relativity.

A deductive argument is more clearly valid or not, which makes them easier to evaluate.

Evaluating deductive arguments

A deductive argument is considered valid if all the premises are true, and the conclusion follows logically from those premises. In other words, the premises are true, and the conclusion follows necessarily from those premises.

Example 19

The argument “All cats are mammals and a tiger is a cat, so a tiger is a mammal” is a valid deductive argument.

The premises are:

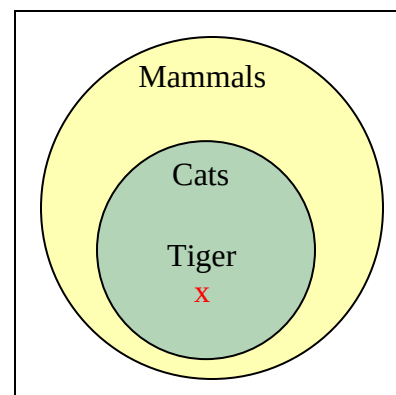
All cats are mammals

A tiger is a cat

The conclusion is:

A tiger is a mammal

Both the premises are true. To see that the premises must logically lead to the conclusion, one approach would be use a Venn diagram. From the first premise, we can conclude that the set of cats is a subset of the set of mammals. From the second premise, we are told that a tiger lies within the set of cats. From that, we can see in the Venn diagram that the tiger also lies inside the set of mammals, so the conclusion is valid.



Analyzing arguments with Venn diagrams¹

To analyze an argument with a Venn diagram

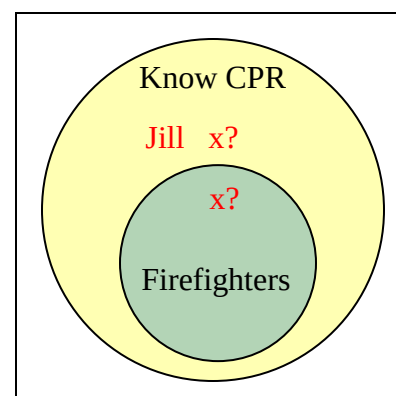
- 1) Draw a Venn diagram based on the premises of the argument
- 2) If the premises are insufficient to determine what determine the location of an element, indicate that.
- 3) The argument is valid if it is clear that the conclusion must be true

Example 20

Premise: All firefighters know CPR
 Premise: Jill knows CPR
 Conclusion: Jill is a firefighter

From the first premise, we know that firefighters all lie inside the set of those who know CPR. From the second premise, we know that Jill is a member of that larger set, but we do not have enough information to know if she also is a member of the smaller subset that is firefighters.

Since the conclusion does not necessarily follow from the premises, this is an invalid argument, regardless of whether Jill actually is a firefighter.



It is important to note that whether or not Jill is actually a firefighter is not important in evaluating the validity of the argument; we are only concerned with whether the premises are enough to prove the conclusion.

Try it Now 2

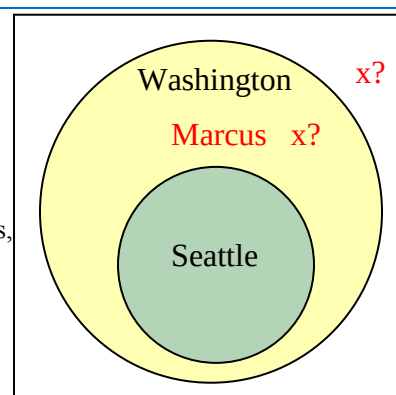
Determine the validity of this argument:

Premise: No cows are purple
 Premise: Fido is not a cow
 Conclusion: Fido is purple

In addition to these categorical style premises of the form “all ____”, “some ____”, and “no ____”, it is also common to see premises that are implications.

Example 21

Premise: If you live in Seattle, you live in Washington.
 Premise: Marcus does not live in Seattle
 Conclusion: Marcus does not live in Washington



¹ Technically, these are Euler circles or Euler diagrams, not Venn diagrams, continue to call them Venn diagrams.

From the first premise, we know that the set of people who live in Seattle is inside the set of those who live in Washington. From the second premise, we know that Marcus does not lie in the Seattle set, but we have insufficient information to know whether or not Marcus lives in Washington or not. This is an invalid argument.

Example 22

Consider the argument “You are a married man, so you must have a wife.”

This is an invalid argument, since there are, at least in parts of the world, men who are married to other men, so the premise not insufficient to imply the conclusion.

Some arguments are better analyzed using truth tables.

Example 23

Consider the argument

Premise: If you bought bread, then you went to the store

Premise: You bought bread

Conclusion: You went to the store

While this example is hopefully fairly obviously a valid argument, we can analyze it using a truth table by representing each of the premises symbolically. We can then look at the implication that the premises together imply the conclusion. If the truth table is a tautology (always true), then the argument is valid.

We’ll get B represent “you bought bread” and S represent “you went to the store”. Then the argument becomes:

Premise: $B \rightarrow S$

Premise: B

Conclusion: S

To test the validity, we look at whether the combination of both premises implies the conclusion; is it true that $[(B \rightarrow S) \wedge B] \rightarrow S$?

B	S	$B \rightarrow S$	$(B \rightarrow S) \wedge B$	$[(B \rightarrow S) \wedge B] \rightarrow S$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the truth table for $[(B \rightarrow S) \wedge B] \rightarrow S$ is always true, this is a valid argument.

Try it Now 3

Determine if the argument is valid:

Premise: If I have a shovel I can dig a hole.
 Premise: I dug a hole
 Conclusion: Therefore I had a shovel

Analyzing arguments using truth tables

To analyze an argument with a truth table:

1. Represent each of the premises symbolically
2. Create a conditional statement, joining all the premises with and to form the antecedent, and using the conclusion as the consequent.
3. Create a truth table for that statement. If it is always true, then the argument is valid.

Example 24

Premise: If I go to the mall, then I'll buy new jeans
 Premise: If I buy new jeans, I'll buy a shirt to go with it
 Conclusion: If I got to the mall, I'll buy a shirt.

Let M = I go to the mall, J = I buy jeans, and S = I buy a shirt.

The premises and conclusion can be stated as:

Premise: $M \rightarrow J$

Premise: $J \rightarrow S$

Conclusion: $M \rightarrow S$

We can construct a truth table for $[(M \rightarrow J) \wedge (J \rightarrow S)] \rightarrow (M \rightarrow S)$

M	J	S	$M \rightarrow J$	$J \rightarrow S$	$(M \rightarrow J) \wedge (J \rightarrow S)$	$M \rightarrow S$	$[(M \rightarrow J) \wedge (J \rightarrow S)] \rightarrow (M \rightarrow S)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the truth table, we can see this is a valid argument.

The previous problem is an example of a syllogism.

Syllogism

A syllogism is an implication derived from two others, where the consequence of one is the antecedent to the other. The general form of a syllogism is:

Premise: $p \rightarrow q$

Premise: $q \rightarrow r$

Conclusion: $p \rightarrow r$

This is sometime called the transitive property for implication.

Example 25

Premise: If I work hard, I'll get a raise.

Premise: If I get a raise, I'll buy a boat.

Conclusion: If I don't buy a boat, I must not have worked hard.

If we let W = working hard, R = getting a raise, and B = buying a boat, then we can represent our argument symbolically:

Premise $H \rightarrow R$

Premise $R \rightarrow B$

Conclusion: $\sim B \rightarrow \sim H$

We could construct a truth table for this argument, but instead, we will use the notation of the contrapositive we learned earlier to note that the implication $\sim B \rightarrow \sim H$ is equivalent to the implication $H \rightarrow B$. Rewritten, we can see that this conclusion is indeed a logical syllogism derived from the premises.

Try it Now 4

Is this argument valid?

Premise: If I go to the party, I'll be really tired tomorrow.

Premise: If I go to the party, I'll get to see friends.

Conclusion: If I don't see friends, I won't be tired tomorrow.

Lewis Carroll, author of *Alice in Wonderland*, was a math and logic teacher, and wrote two books on logic. In them, he would propose premises as a puzzle, to be connected using syllogisms.

Example 26

Solve the puzzle. In other words, find a logical conclusion from these premises.

All babies are illogical.

Nobody is despised who can manage a crocodile.

Illogical persons are despised.

Let B = is a baby, D = is despised, I = is illogical, and M = can manage a crocodile.

Then we can write the premises as:

$B \rightarrow I$

$M \rightarrow \sim D$

$I \rightarrow D$

From the first and third premises, we can conclude that $B \rightarrow D$; that babies are despised. Using the contrapositive of the second premise, $D \rightarrow \sim M$, we can conclude that $B \rightarrow \sim M$; that babies cannot manage crocodiles.

While silly, this is a logical conclusion from the given premises.

Logical Fallacies in Common Language

In the previous discussion, we saw that logical arguments can be invalid when the premises are not true, when the premises are not sufficient to guarantee the conclusion, or when there are invalid chains in logic. There are a number of other ways in which arguments can be invalid, a sampling of which are given here.

Ad hominem

An ad hominem argument attacks the person making the argument, ignoring the argument itself.

Example 27

“Jane says that whales aren’t fish, but she’s only in the second grade, so she can’t be right.”

Here the argument is attacking Jane, not the validity of her claim, so this is an ad hominem argument.

Example 28

“Jane says that whales aren’t fish, but everyone knows that they’re really mammals – she’s so stupid.”

This certainly isn’t very nice, but it is *not* ad hominem since a valid counterargument is made along with the personal insult.

Appeal to ignorance

This type of argument assumes something is true because it hasn’t been proven false.

Example 29

“Nobody has proven that photo isn’t Bigfoot, so it must be Bigfoot.”

Appeal to authority

These arguments attempt to use the authority of a person to prove a claim. While often authority can provide strength to an argument, problems can occur when the person’s opinion is not shared by other experts, or when the authority is irrelevant to the claim.

Example 30

“A diet high in bacon can be healthy – Doctor Atkins said so.”

Here, an appeal to the authority of a doctor is used for the argument. This generally would provide strength to the argument, except that the opinion that eating a diet high in saturated fat runs counter to general medical opinion. More supporting evidence would be needed to justify this claim.

Example 31

“Jennifer Hudson lost weight with Weight Watchers, so their program must work.”

Here, there is an appeal to the authority of a celebrity. While her experience does provide evidence, it provides no more than any other person’s experience would.

Appeal to consequence

An appeal to consequence concludes that a premise is true or false based on whether the consequences are desirable or not.

Example 32

“Humans will travel faster than light: faster-than-light travel would be beneficial for space travel.”

False dilemma

A false dilemma argument falsely frames an argument as an “either or” choice, without allowing for additional options.

Example 33

“Either those lights in the sky were an airplane or aliens. There are no airplanes scheduled for tonight, so it must be aliens.”

This argument ignores the possibility that the lights could be something other than an airplane or aliens.

Circular reasoning

Circular reasoning is an argument that relies on the conclusion being true for the premise to be true.

Example 34

“I shouldn’t have gotten a C in that class; I’m an A student!”

In this argument, the student is claiming that because they’re an A student, though shouldn’t have gotten a C. But because they got a C, they’re not an A student.

Straw man

A straw man argument involves misrepresenting the argument in a less favorable way to make it easier to attack.

Example 35

“Senator Jones has proposed reducing military funding by 10%. Apparently he wants to leave us defenseless against attacks by terrorists”

Here the arguer has represented a 10% funding cut as equivalent to leaving us defenseless, making it easier to attack.

Post hoc (post hoc ergo propter hoc)

A post hoc argument claims that because two things happened sequentially, then the first must have caused the second.

Example 36

“Today I wore a red shirt, and my football team won! I need to wear a red shirt every time they play to make sure they keep winning.”

Correlation implies causation

Similar to post hoc, but without the requirement of sequence, this fallacy assumes that just because two things are related one must have caused the other. Often there is a third variable not considered.

Example 37

“Months with high ice cream sales also have a high rate of deaths by drowning. Therefore ice cream must be causing people to drown.”

This argument is implying a causal relation, when really both are more likely dependent on the weather; that ice cream and drowning are both more likely during warm summer months.

Try it Now 5

Identify the logical fallacy in each of the arguments

- Only an untrustworthy person would run for office. The fact that politicians are untrustworthy is proof of this.
-

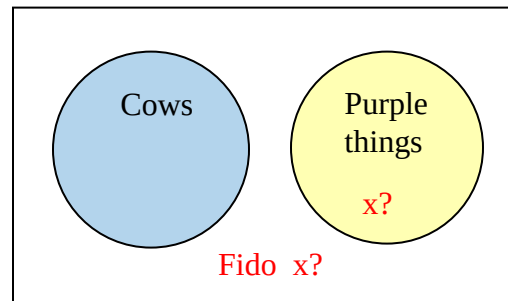
- b. Since the 1950s, both the atmospheric carbon dioxide level and obesity levels have increased sharply. Hence, atmospheric carbon dioxide causes obesity.
 - c. The oven was working fine until you started using it, so you must have broken it.
 - d. You can't give me a D in the class – I can't afford to retake it.
 - e. The senator wants to increase support for food stamps. He wants to take the taxpayers' hard-earned money and give it away to lazy people. This isn't fair so we shouldn't do it.
-

Try it Now Answers

1.

A	B	$\sim A$	$\sim A \wedge B$	$\sim B$	$(\sim A \wedge B) \vee \sim B$
T	T	F	F	F	F
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	F	T	T

2. Since no cows are purple, we know there is no overlap between the set of cows and the set of purple things. We know Fido is not in the cow set, but that is not enough to conclude that Fido is in the purple things set.



3. Let S : have a shovel, D : dig a hole

The first premise is equivalent to $S \rightarrow D$. The second premise is D . The conclusion is S .

We are testing $[(S \rightarrow D) \wedge D] \rightarrow S$

S	D	$S \rightarrow D$	$(S \rightarrow D) \wedge D$	$[(S \rightarrow D) \wedge D] \rightarrow S$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

This is not a tautology, so this is an invalid argument.

4. Letting P = go to the party, T = being tired, and F = seeing friends, then we can represent this argument as P :

Premise: $P \rightarrow T$

Premise: $P \rightarrow F$

Conclusion: $\sim F \rightarrow \sim T$

We could rewrite the second premise using the contrapositive to state $\sim F \rightarrow \sim P$, but that does not allow us to form a syllogism. If we don't see friends, then we didn't go the party, but that is not sufficient to claim I won't be tired tomorrow. Maybe I stayed up all night watching movies.

5.
 - a. Circular
 - b. Correlation does not imply causation
 - c. Post hoc
 - d. Appeal to consequence
 - e. Straw man
-

Exercises

Compute the truth values of the following symbolic statements, supposing that the truth value of A, B, C is T, and the truth value of X, Y, Z is F.

1. $\sim A \vee B$
2. $\sim B \vee X$
3. $\sim Y \vee C$
4. $\sim Z \vee X$
5. $(A \wedge X) \vee (B \wedge Y)$
6. $(B \wedge C) \vee (Y \wedge Z)$
7. $\sim(C \wedge Y) \vee (A \wedge Z)$
8. $\sim(A \wedge B) \vee (X \wedge Y)$
9. $\sim(X \wedge Z) \vee (B \wedge C)$
10. $\sim(X \wedge \sim Y) \vee (B \wedge \sim C)$
11. $(A \vee X) \wedge (Y \vee B)$
12. $(B \vee C) \wedge (Y \vee Z)$
13. $(X \vee Y) \wedge (X \vee Z)$
14. $\sim(A \vee Y) \wedge (B \vee X)$
15. $\sim(X \vee Z) \wedge (\sim X \vee Z)$
16. $\sim(A \vee C) \vee \sim(X \wedge \sim Y)$
17. $\sim(B \vee Z) \wedge \sim(X \vee \sim Y)$
18. $\sim[(A \vee \sim C) \vee (C \vee \sim A)]$
19. $\sim[(B \wedge C) \wedge \sim(C \wedge B)]$
20. $\sim[(A \wedge B) \vee \sim(B \wedge A)]$
21. $[A \vee (B \vee C)] \wedge \sim[(A \vee B) \vee C]$
22. $[X \vee (Y \wedge Z)] \vee \sim[(X \vee Y) \wedge (X \vee Z)]$
23. $[A \wedge (B \vee C)] \wedge \sim[(A \wedge B) \vee (A \wedge C)]$
24. $\sim\{[(\sim A \wedge B) \wedge (\sim X \wedge Z)] \wedge \sim[(A \wedge \sim B) \vee (\sim Y \wedge \sim Z)]\}$
25. $\sim\{\sim[(B \wedge \sim C) \vee (Y \wedge \sim Z)] \wedge [(\sim B \vee X) \vee (B \vee \sim Y)]\}$
26. $A \rightarrow B$
27. $A \rightarrow X$
28. $B \rightarrow Y$
29. $Y \rightarrow Z$

30. $(A \rightarrow B) \rightarrow Z$
31. $(X \rightarrow Y) \rightarrow Z$
32. $(A \rightarrow B) \rightarrow C$
33. $(X \rightarrow Y) \rightarrow C$
34. $A \rightarrow (B \rightarrow Z)$
35. $X \rightarrow (Y \rightarrow Z)$
36. $[(A \rightarrow B) \rightarrow C] \rightarrow Z$
37. $[(A \rightarrow X) \rightarrow Y] \rightarrow Z$
38. $[A \rightarrow (X \rightarrow Y)] \rightarrow C$
39. $[A \rightarrow (B \rightarrow Y)] \rightarrow X$
40. $[(X \rightarrow Z) \rightarrow C] \rightarrow Y$
41. $[(Y \rightarrow B) \rightarrow Y] \rightarrow Y$
42. $[(A \rightarrow Y) \rightarrow B] \rightarrow Z$
43. $[(A \wedge X) \rightarrow C] \rightarrow [(X \rightarrow C) \rightarrow X]$
44. $[(A \wedge X) \rightarrow C] \rightarrow [(A \rightarrow X) \rightarrow C]$
45. $[(A \wedge X) \rightarrow Y] \rightarrow [(X \rightarrow A) \rightarrow (A \rightarrow Y)]$
46. $[(A \wedge X) \vee (\sim A \wedge \sim X)] \rightarrow [(A \rightarrow X) \wedge (X \rightarrow A)]$
47. $\{[A \rightarrow (B \rightarrow C)] \rightarrow [(A \wedge B) \rightarrow C]\} \rightarrow [(Y \rightarrow B) \rightarrow (C \rightarrow Z)]$
48. $\{[(X \rightarrow Y) \rightarrow Z] \rightarrow [Z \rightarrow (X \rightarrow Y)]\} \rightarrow [(X \rightarrow Z) \rightarrow Y]$
49. $[(A \wedge X) \rightarrow Y] \rightarrow [(A \rightarrow X) \wedge (A \rightarrow Y)]$
50. $[A \rightarrow (X \wedge Y)] \rightarrow [(A \rightarrow X) \vee (A \rightarrow Y)]$

Construct the complete truth table for each of the following formulas:

51. $(P \wedge Q) \vee (P \wedge \sim Q)$
52. $\sim(P \wedge \sim P)$
53. $\sim(P \vee \sim P)$
54. $\sim(P \wedge Q) \vee (\sim P \vee \sim Q)$
55. $\sim(P \vee Q) \vee (\sim P \wedge \sim Q)$
56. $(P \wedge Q) \vee (\sim P \wedge \sim Q)$
57. $\sim(P \vee (P \wedge Q))$
58. $\sim(P \vee (P \wedge Q)) \vee P$
59. $(P \wedge (Q \vee P)) \wedge \sim P$
60. $((P \rightarrow Q) \rightarrow P) \rightarrow P$
61. $\sim(\sim(P \rightarrow Q) \rightarrow P)$
62. $(P \rightarrow Q) \leftrightarrow \sim P$
63. $P \rightarrow (Q \rightarrow (P \wedge Q))$
64. $(P \vee Q) \leftrightarrow (\sim P \rightarrow Q)$
65. $\sim(P \vee (P \rightarrow Q))$
66. $(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$
67. $(P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P)$
68. $(P \vee Q) \rightarrow (P \wedge Q)$
69. $(P \wedge Q) \vee (P \wedge R)$
70. $[P \leftrightarrow (Q \leftrightarrow R)] \leftrightarrow [(P \leftrightarrow Q) \leftrightarrow R]$
71. $[P \rightarrow (Q \wedge R)] \rightarrow [P \rightarrow R]$
72. $[P \rightarrow (Q \vee R)] \rightarrow [P \rightarrow Q]$
73. $[(P \rightarrow Q) \rightarrow R] \rightarrow [P \rightarrow R]$

$$74. [(P \wedge Q) \rightarrow R] \rightarrow [P \rightarrow R]$$

$$75. [(P \wedge Q) \rightarrow R] \rightarrow [(Q \wedge \sim R) \rightarrow \sim P]$$